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Stimulated Emission of Photon Excitations by External Currents in Spacetime

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The problem of stimulated emission of photon excitations by external currents is studied in *spacetime* by making use of the concept of localized photon excitations in configuration space. An explicit expression is derived for the amplitude that an arbitrary number of photon excitations are produced and found in arbitrary localized regions in space when there are an arbitrary number of photon excitations prior to the switching on of the intervening current. Considered as an application is the reaction of a "photon splitting" to any number of photon excitations as the latter emerge spatially within a cone in the presence of a strong external electromagnetic current. This work is a generalization of work dealing with strictly massive particles.

1. INTRODUCTION

Many experiments [see, e.g., Franson and Potocki (1988) and Grangier *et al.* (1986), tracing back to the pioneering work of Taylor (1909)] give a clear indication of the localization of photons by detectors. As a generalization of other work (Manoukian, 1989), I investigate the problem of stimulated emission of photon excitations, by external electromagnetic currents, as they are observed in *spacetime* in conformity with experiments. This is done by making use of the concept of localized photon excitations in configuration space (Manoukian, 1988). I derive an explicit expression for the amplitude that an arbitrary number of photon excitations are produced by an intervening current distribution, as the former are found in various localized regions of space, when there are initially an arbitrary number of photon excitations *prior to* the switching on of the current in question. As an application, I consider the process of "photon splitting" to any number of photon excitations as the latter merge spatially into a cone in the presence of a strong electromagnetic current.

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2. STIMULATED EMISSION OF PHOTON EXCITATIONS IN SPACETIME

Our starting point is the vacuum-to-vacuum transition amplitude (Schwinger, 1970, 1977) for photons in the presence of an external electromagnetic current $J^u(x)$. In the Coulomb gauge, the latter is given by the expression (cf. Manoukian, 1986)

$$\langle 0_+ | 0_- \rangle^J = \exp[iW] \tag{1}$$

$$W = \frac{1}{2} \int (dx)\,(dx') \left[J^0(x) \frac{1}{\boldsymbol{\partial}^2} \delta(x-x') J^0(x') + J^i(x) \left(\delta^{ij} - \frac{\partial^i \partial^j}{\boldsymbol{\partial}^2} \right) D_+(x-x') J^j(x') \right] \tag{2}$$

where

$$D_+(x-x') = \int \frac{(dk)}{(2\pi)^4} \frac{e^{ik(x-x')}}{k^2 - i\varepsilon}, \qquad \varepsilon \to +0 \tag{3}$$

It is convenient to define

$$J_T^i(x) = \left(\delta^{ij} - \frac{\partial^i \partial^j}{\boldsymbol{\partial}^2} \right) J^i(x) \tag{4}$$

and note that $\partial_i J_T^i(x) = 0$; $i, j = 1, 2, 3$.

Let $\mathbf{n}$ be any (three-dimensional) unit vector and define (Manoukian, 1988)

$$\mathbf{j}(x) = \int \frac{d^3\mathbf{k}}{(2\pi)^3 (2k^0)^{1/2}} \left[J_T(k) - \left(\frac{\mathbf{k} + k^0 \mathbf{n}}{k^0 + \mathbf{n} \cdot \mathbf{k}} \right) \mathbf{n} \cdot \mathbf{J}_T(k) \right] e^{ikx} \tag{5}$$

$$x = (x^0, \mathbf{x}), \qquad k^0 = |\mathbf{k}|, \qquad J_T(k) = \int (dx)\, e^{-ikx} J_T(x) \tag{6}$$

Note in particular that

$$\mathbf{n} \cdot \mathbf{j}(x) = 0 \tag{7}$$

for *all* x. We introduce two polarization (unit) vectors $\boldsymbol{\varepsilon}_1$, $\boldsymbol{\varepsilon}_2$ such that

$$\boldsymbol{\varepsilon}_\lambda \cdot \boldsymbol{\varepsilon}_{\lambda'} = \delta_{\lambda\lambda'}, \qquad \mathbf{n} \cdot \boldsymbol{\varepsilon}_\lambda = 0, \qquad \lambda = 1, 2 \tag{8}$$

and write in coordinate space a completeness relation:

$$\delta^{ij} = n^i n^j + \sum_\lambda \varepsilon_\lambda^i \varepsilon_\lambda^j \tag{9}$$

Finally we define the objects

$$a_\lambda(x) = \boldsymbol{\varepsilon}_\lambda \cdot \mathbf{j}(x) \tag{10}$$

defining only *two* degrees of freedom for photon excitations at each point of spacetime.

The vacuum persistence probability may then be written as

$$|\langle 0_+|0_-\rangle|^2 = \exp\left[-\sum_{\lambda=1,2}\int d^3\mathbf{x}\,|a_\lambda(x)|^2\right] \tag{11}$$

To obtain the amplitudes for stimulated emission, we proceed as follows. We write $J^\mu = J_1^\mu + J_2^\mu + J_3^\mu$, where the (intervening) current J_2^μ is switched on after the current J_1^μ is switched off, and the current J_3^μ is switched on after the current J_2^μ is switched off. After straightforward manipulations, we may rewrite (1) as

$$\begin{aligned}\langle 0_+|0_-\rangle^J = {} & \langle 0_+|0_-\rangle^{J_1}\cdot\langle 0|0_-\rangle^{J_2}\cdot\langle 0_+|0_-\rangle^{J_3}\\ & \times\exp[A_{32}]\exp[A'_{31}]\exp[A_{21}]\end{aligned} \tag{12}$$

where

$$A_{32} = \sum_\alpha (i)a_\alpha^{3*}(i)a_\alpha^2 \tag{13}$$

$$A_{21} = \sum_\alpha (i)a_\alpha^{2*}(i)a_\alpha^1 \tag{14}$$

$$A'_{31} = \sum_{\alpha,\alpha'} (i)a_\alpha^{3*}\tilde{\delta}_{\alpha\alpha'}(i)a_{\alpha'}^1 \tag{15}$$

$\alpha = (y, \lambda)$ in a convenient discrete (Schwinger, 1970; Manoukian, 1988) space variable notation (a *lattice*) at a given time by defining

$$a_\alpha = (d^3\mathbf{y})^{1/2}a_\lambda(y) \tag{16}$$

and

$$\tilde{\delta}_{\alpha\alpha'} = (d^3\mathbf{y}\,d^3\mathbf{y}')^{1/2}\int d^3\mathbf{x}\left[\Delta(y-x)i\frac{\overleftrightarrow{\partial}}{\partial x^0}\Delta(x-y')\right] \tag{17}$$

$$y^0 > x^0 > y'^0, \qquad \Delta(y-x) = \int\frac{d^3\mathbf{k}}{(2\pi)^3(2k^0)^{1/2}}\,e^{ik(y-x)} \tag{18}$$

$\overleftrightarrow{\partial}/\partial x^0 = \overrightarrow{\partial}/\partial x^0 - \overleftarrow{\partial}/\partial x^0$. We also carry out a unitarity expansion of $\langle 0_+|0_-\rangle^J$ in configuration space:

$$\begin{aligned}\langle 0_+|0_-\rangle^J = {} & \sum\nolimits_* \langle 0_+|N; N_1, N_2, \ldots, y_2^0\rangle^{J_3}\\ & \times\langle N; N_1, N_2, \ldots, y_2^0|M; M_1, M_2, \ldots, y_1^0\rangle^{J_2}\\ & \times\langle M; M_1, M_2, \ldots, y_1^0|0_-\rangle^{J_1}\end{aligned} \tag{19}$$

where $y_2^0 > y_1^0$; $\langle M; M_1, M_2, \ldots, y_1^0|0_-\rangle^{J_1}$ denotes the amplitude that M photon excitations are emitted by the current J_1^μ, M_1 of which are found at lattice site $\alpha_1=(y_1^0, \mathbf{y}_1, \lambda_1)$, M_2 of which are found at $\alpha_2=(y_1^0, \mathbf{y}_2, \lambda_2)$, and so on, *at a time* y_1^0 after the current J_1^μ ceases to operate here

$$\langle N; N_1, N_2, \ldots, y_2^0 | M; M_1, M_2, \ldots, y_1^0\rangle^{J_2}$$

denotes the amplitude that the M photon excitations move in the presence of the *intervening* current, and at a later time y_2^0, after the latter current ceases to operate, we find N photon excitations, N_1 of which are at lattice site $\alpha_1'=(y_2^0, \mathbf{y}_1', \lambda_1)$, N_2 of which are at lattice site $\alpha_1'=(y_2^0, \mathbf{y}_2', \lambda_2)$, and so on; finally, $\langle 0_+ | N; N_1, N_2, \ldots, y_2^0\rangle^{J_3}$ is the amplitude that the N photon excitations are finally detected by the current J_3.

Upon expanding the exponentials in (12) (Manoukian, 1988) and comparing the resulting expression with (19), we arrive at the expression for the amplitude of stimulated emission of photon excitations:

$$\begin{aligned}
&\langle N; N_1, N_2, \ldots, y_2^0 | M; M_1, M_2, \ldots, y_1^0\rangle^{J} \\
&\quad = \langle 0_+ | 0_-\rangle^J (N_1!\, N_2! \ldots M_1!\, M_2! \ldots)^{1/2} \\
&\quad \times \sum{}^* \frac{(ia_{\alpha_1'})^{N_1-m_1}}{(N_1-m_1)!} \frac{(ia_{\alpha_2'})^{N_2-m_2}}{(N_2-m_2)!} \\
&\quad \times \frac{(\tilde{\delta}_{\alpha_1'\alpha_1})^{m_{11}}}{m_{11}!} \frac{(\tilde{\delta}_{\alpha_1'\alpha_2})^{m_{12}}}{m_{12}!} \cdots \frac{(\tilde{\delta}_{\alpha_2'\alpha_1})^{m_{21}}}{m_{21}!} \frac{(\tilde{\delta}_{\alpha_2'\alpha_2})^{m_{22}}}{m_{22}!} \\
&\quad \times \cdots \frac{(ia^*_{\alpha_1})^{m_1-\sum_i m_{i1}}}{(M_1-\sum_i m_{i1})!} \frac{(ia^*_{\alpha_2})^{m_2-\sum_i m_{i2}}}{(M_2-\sum_i m_{i2})!} \cdots
\end{aligned} \tag{20}$$

where $\alpha_1=(y_2^0, \mathbf{y}_1, \lambda_1)$, $\alpha_2=(y_2^0, \mathbf{y}_2, \lambda_2), \ldots$, $\alpha_1'=(y_1^0, \mathbf{y}_1', \lambda_1')$, $\alpha_2'=(y_1^0, \mathbf{y}_2', \lambda_2'), \ldots$, and $\sum^*$ stands for a summation over all nonnegative integers $m_1, m_2, \ldots, m$, $m_{11}, m_{12}, \ldots, m_{21}$, $m_{22}, \ldots$, satisfying the constraints

$$\begin{aligned}
&m_{11}+m_{12}+\cdots=m_1, \qquad 0<\sum_i m_{i1}\leq M_1 \\
&m_{21}+m_{22}+\cdots=m_2, \qquad 0\leq\sum_i m_{i2}\leq M_2 \\
&\qquad\vdots \qquad\qquad\qquad\qquad \vdots \\
&\qquad\qquad 0\leq m_1\leq N_1 \\
&\qquad\qquad 0\leq m_2\leq N_2 \\
&\qquad\qquad\quad \vdots \\
&m_1+m_2+\cdots=m
\end{aligned} \tag{21}$$

$y_2^0>y_1^0$, and the time of operation of the current J^μ is between y_1^0 and y_2^0.

The expression (20) is quite general, an explicit application of which is given in the next section.

3. "PHOTON SPLITTING" AS A STIMULATED EMISSION

We are interested in the process of a "photon splitting" into any number of photon excitations with arbitrary polarizations λ as the latter move into a cone $C: \mathbf{x}=(|\mathbf{x}|, \theta, \phi)$, $0\leq|\mathbf{x}|<\infty$, $\theta_0\leq\theta\leq\theta_0+\Delta\theta$, $\phi_0\leq\phi\leq\phi_0+\Delta\phi$, in the presence of a strong electromagnetic current J^μ. To this end the connected process corresponding to this is read from (20) to be

$$\langle N; N_1, N_2, \ldots, y_2^0|1_{\alpha_1'}, y_1^0\rangle^J = \langle 0_+|0_-\rangle^J (ia_{\alpha_1'}) \frac{(ia^*_{\alpha_1})^{N_1}}{(N_1!)^{1/2}} \frac{(ia^*_{\alpha_2})^{N_2}}{(N_2!)^{1/2}} \cdots \tag{22}$$

In a convenient lattice space notation we write $C=\{\mathbf{x}_1, \mathbf{x}_2, \ldots\}$. Then the transition probability for the process in question may be written as

$$|\langle 0_+|0_-\rangle^J|^2 |a_{\alpha_1'}|^2 \sum_{N=0}^{\infty} \sum_{N_1+N_2+\cdots=N} \frac{|a_{\alpha_1}|^2}{N_1!} \frac{|a_{\alpha_2}|^2}{N_2!} \cdots \tag{23}$$

where $\alpha_1=(y_2^0, \mathbf{x}_1, \lambda_1)$, $\alpha_2=(y_2^0, \mathbf{x}_2, \lambda_2), \ldots, \alpha_1'=(y_1^0, \mathbf{y}', \lambda')$. The summation in (23) may be explicitly carried out in the sense of a multinomial expansion to give, in a continuous space variable notation, for the probability of occurrence of the process in question

$$d^3y'|a_{\lambda'}(y_1^0, \mathbf{y}_1')|^2 \exp\left[\sum_\lambda \int_C d^3\mathbf{x}|a_\lambda(x)|^2\right] \exp\left[-\sum_\lambda \int_{R^3} d^3x|a_\lambda(x)|^2\right] \tag{24}$$

where $x^0\equiv y_2^0$. Obviously the probability in (24) is time (y_2^0) dependent. Other stimulated emission processes are similarly handled.

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